Alpha-Nucleus Potential and the Structure of the Alpha-Particle

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In the first order approximation, it is shown that the inclusion of higher shell model orbitals in the intrinsic wave function of the alpha particle gives alpha-nucleus potentials of the Woods-Saxon type with the same radius and diffusity as for typical nucleon-nucleus potentials and with a depth equal to the sum of the four nucleon-nucleus depths. We concluded that the alpha-nucleus potentials must have the same ranges for radius and diffusity as the nucleon-nucleus potentials.

It is well known that there are many ambiguities in the determination of the optical model parameters for the alpha particles. One way to determine them is to study the range of each parameter, used in the analysis of the alpha particle elastic scattering for a large region of masses and energies, which gives equally good fit to the experimental data. Unfortunately, such a detailed analysis has not been done. Another way of tackling the problem is to calculate the alpha-nucleus potential from known nucleon-nucleus potentials. Although this cannot be done very accurately, it may give some indications to the range of the parameters which have to be used in the fitting.

The first order alpha-nucleus potential is given by the expression

$$V_\alpha(R) = \langle \Phi_\alpha | \sum_{i=1}^{4} V_i^{\text{opt}}(r_i) \, | \Phi_\alpha \rangle \qquad (1)$$

where Φ_α is the ground state of the alpha particle and $V_i^{\text{opt}}(r_i)$ the real part of the nucleon-nucleus optical potentials.

These kinds of calculations where first performed for alphas in Reference [1]. Assuming a Gaussian alpha particle internal wave function and Perey's set of parameters for the nucleon-nucleus potentials [2] it has been shown that the expression (1) can be fitted very accurately by a Woods-Saxon form with approximately the same radius as for typical nucleon-nucleus potentials, with the diffusity increased by 30% and with the depth equal to the sum of the nucleon depths.

Recent analysis of electron scattering by helium nuclei for the form factor of the alpha particle suggests that higher orbitals be included in the description of the alpha particle [3]. For this purposes Moshinsky et al. [3] have derived a complete set of four-particle nonspurious harmonic oscillator states $|n_1 l_1, n_2 l_2, n_3 l_3\rangle$ that are symmetric under the exchange of the nucleons and have total orbital angular momentum zero. For an expansion of up to four quanta

$$N = \sum_{i=1}^{3} 2 n_i + l_i = 4 \,,$$

it has been shown that a good fit to the experimental values can be obtained by the following linear combination of harmonic-oscillator states

$$\Phi_\alpha = \cos\gamma \, |1\rangle + \sin\gamma \, |4\rangle \qquad (2)$$

where

$$|1\rangle = |00, 00, 00\rangle \qquad (3)$$

$$|4\rangle = \frac{1}{2\sqrt{2}} |20, 00, 00\rangle + \frac{1}{2\sqrt{2}} |00, 20, 00\rangle + \frac{2}{3\sqrt{2}} |00, 00, 20\rangle + \frac{1}{6}\sqrt{\frac{5}{3}} \, |10,10, 00\rangle$$

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$$+\frac{1}{3\sqrt{3}}\,|02,02,00\rangle+\frac{1}{3}\sqrt{\frac{5}{3}}\,|10,01,01\rangle$$
$$+\frac{2}{3\sqrt{3}}\,|02,01,01\rangle-\frac{1}{3}\,|00,11,01\rangle \tag{4}$$

with $\gamma = -30°$ and $\hbar\omega = 33.6$ MeV or $\gamma = -70°$ and $\hbar\omega = 37$ MeV.

In the present note we show that the first order alpha-nucleus potential obtained with the help of expression (1) where Φ_α is the improved intrinsic wave function (2) can be very accurately fitted by a Woods-Saxon potential with the same radius and diffusity as for the used nucleon-nucleus potentials and a depth equal with the sum of the depths of the nucleon-nucleus potentials acting on the constituent nucleons.

Indeed, taking into account that the radius of the alpha particles is much smaller than the radius of heavy nuclei, we can expand the nucleon-nucleus potentials in terms of powers of the intrinsic coordinates of the alpha particle [4].

Keeping terms up to second order we have

$$\sum_{i=1}^{4} V_i(r_i) = V_{\text{on}}[f(r_1)+f(r_2)] + V_{\text{op}}[f(r_3)+f(r_4)]$$
$$= 2(V_{\text{on}}+V_{\text{op}})\,f(R) + \frac{V_{\text{on}}-V_{\text{op}}}{\sqrt{\alpha}}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R}\cdot\left[\sqrt{\frac{2}{3}}\,\xi_2\,P_1(\boldsymbol{R}\hat{\boldsymbol{\xi}}_2)+\frac{1}{\sqrt{3}}\,\xi_3\,P_1(\boldsymbol{R}\hat{\boldsymbol{\xi}}_3)\right]$$
$$+\frac{V_{\text{on}}-V_{\text{op}}}{\alpha\sqrt{18}}\left[\frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2}-\frac{1}{R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R}\right]\cdot\xi_2\,\xi_3\,P_1(\boldsymbol{R}\hat{\boldsymbol{\xi}}_2)\,P_1(\boldsymbol{R}\hat{\boldsymbol{\xi}}_3)$$
$$+\frac{1}{\alpha R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R}\,\frac{1}{\sqrt{18}}\,\xi_2\,\xi_3\,P_1(\boldsymbol{\xi}_2\hat{\boldsymbol{\xi}}_3)$$
$$+\frac{1}{\alpha}\left[\frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2}-\frac{1}{R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R}\right]\cdot\left[\frac{V_{\text{on}}}{3}\,\xi_1^2\,P_2(\boldsymbol{R}\hat{\boldsymbol{\xi}}_1)+\frac{V_{\text{on}}+2\,V_{\text{op}}}{9}\cdot\xi_2^2\,P_2(\boldsymbol{R}\hat{\boldsymbol{\xi}}_2)+\frac{V_{\text{on}}+5\,V_{\text{op}}}{18}\,\xi_3^2\,P_2(\boldsymbol{R}\hat{\boldsymbol{\xi}}_3)\right]$$
$$+\frac{1}{\alpha}\left[\frac{1}{2}\,\frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2}+\frac{1}{R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R}\right]\cdot\left[\frac{V_{\text{on}}}{3}\,\xi_1^2+\frac{V_{\text{on}}+2\,V_{\text{op}}}{9}\,\xi_2^2+\frac{V_{\text{on}}+5\,V_{\text{op}}}{18}\,\xi_3^2\right] \tag{5}$$

where $\boldsymbol{\xi}_i$ are the Jacobi coordinates

$$\begin{aligned}
\boldsymbol{\xi}_1 &= \frac{\alpha}{\sqrt{2}}\,(\boldsymbol{r}_1-\boldsymbol{r}_2)\,,\\
\boldsymbol{\xi}_2 &= \frac{\alpha}{\sqrt{2}}\,(\boldsymbol{r}_1+\boldsymbol{r}_2-3\,\boldsymbol{r}_3)\,,\\
\boldsymbol{\xi}_3 &= \frac{\alpha}{\sqrt{12}}\,(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3-3\,\boldsymbol{r}_4)\,,\\
\boldsymbol{R} &= \frac{\alpha}{2}\,(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4)\,.
\end{aligned} \tag{6}$$

$P_l(\cos\Theta)$ the Legendre polynomials and

$$V_i(r_i) = V_{0i}\,f(r_i) = V_{0i}\,\frac{1}{1+\exp\{r_i - R/a\}} \tag{7}$$

the Woods-Saxon potentials with the following parameters: depth for protons $V_{\text{op}} = -50$ MeV, depth for neutrons $V_{\text{on}} = -60$ MeV, radius of nucleus $R = r_0 A^{1/3}$ with $r_0 = 1.25$ fm and diffusity $a = 0.59$ fm.

Integrating on the internal coordinates of the alpha particle we obtain for the alpha-nucleus potentials the following expressions:

$$V_{\mathrm{G}}(R) = 220\,f(R) + 82.5\,\frac{1}{\alpha R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R} + 41.25\,\frac{1}{\alpha}\,\frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2} \tag{8}$$

for the Gauss wave function, with $\alpha = 0.434\ \mathrm{fm}^{-2}$ and

$$V_{\mathrm{M}}(R) = 220\,f(R) + (82.5\cos^2\gamma + 102.22\sin^2\gamma)\cdot\frac{1}{\alpha R}\,\frac{\mathrm{d}f(R)}{\mathrm{d}R} + (41.25\cos^2\gamma + 58.01\sin^2\gamma)\,\frac{1}{\alpha}\,\frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2} \tag{9}$$

with $\gamma = -30°$, $\hbar\omega = 33.6$ MeV or $\gamma = -70°$, $\hbar\omega = 37$ MeV for the Moshinsky wave function (2).

The above potentials are illustrated in Fig. 1 where for comparison a Woods-Saxon potential with the same radius $r_0 = 1.25$ fm and diffusity $a = 0.59$ fm as the nucleon-potentials but a different depth $V_0 = -198$ MeV is given. We can see that the computed Moshinsky alpha-nucleus potential is very similar with the choosen Woods-Saxon potential. For both sets of parameters γ, $\hbar\omega$ we obtained the same alpha-nucleus potential.

In order to compare the above potentials more fully, we also computed the barrier penetrabilities for alpha decay of ^{212}Po. It is well known that the penetrabilities are very sensible to the parameters

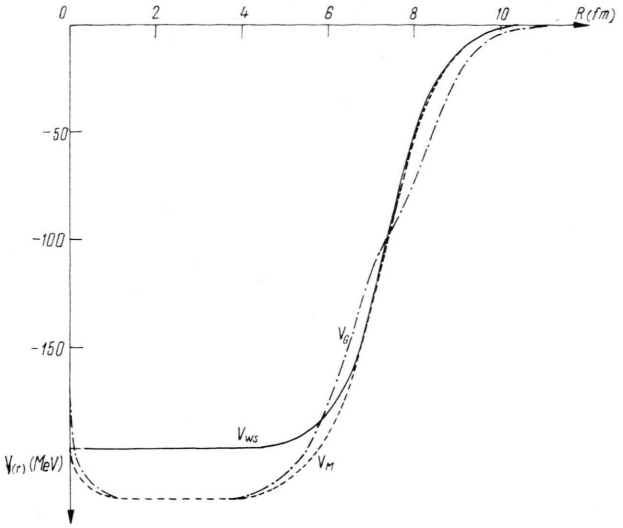

Fig. 1. First order alpha-nucleus potentials V_{G} and V_{M}, obtained by averaging the sum of the four nucleon-nucleus potentials with a Gauss respectively a Moshinsky intrinsic wave function, and a Woods-Saxon potential V_{WS} ($r_0 = 1.25$ fm, $a = 0.59$ fm, $V_0 = -198$ MeV) with the same radius and diffusity as the typical nucleon-nucleus potentials.

of the alpha-nucleus potentials. For the penetrabilities we used the definition

$$P_l(\varepsilon) = 2kR\,|\varphi_l(\infty)/\varphi_l(R)|^2 \tag{10}$$

where l is the angular momentum of the alpha-daughter nucleus relative motion, k the wave number and $\varphi_l(R)$ the relative motion wave function satisfying purely outgoing asymptotic boundary conditions. The calculation of the penetrability (10) was performed by numerical integration of the Schödinger equation [5] and the results are represented in Figure 2.

From these figures we can see that the inclusion of higher shell model orbitals in the alpha particle

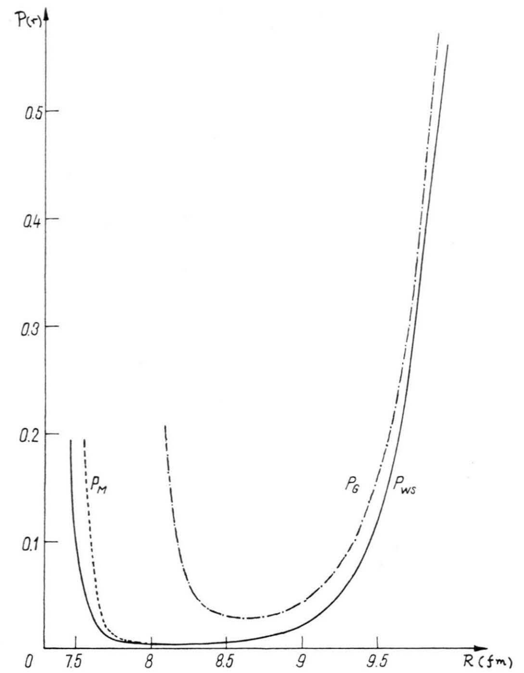

Fig. 2. The penetrabilities P_{G}, P_{M} and P_{WS} as functions of the channel radius R for the corresponding V_{G}, V_{M} and V_{WS} alpha-nucleus potentials.

intrinsic wave function gives first order alpha-nucleus potentials with the same radius and diffusity as the nucleon-nucleus potentials.

These results lead to the conclusion that alpha scattering data should be well fitted using the same geometrical parameters for the real part of the optical alpha-nucleus potential as for the nucleon-nucleus one (e.g. the parameters obtained in [6]). More than that, in the R matrix theory of alpha decay, the use of such parameters especially the smaller radius, gives decay constants not depending on the channel radius [7].

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